A passive compliant Gough-Whitehall-Stewart mechanism for peg-hole disassembly : The Approximation Model

As mentioned, the new device is a modified version of the compliant device in (McCallion, et al., 1979), which was inspired by the Gough-Whitehall-Stewart mechanism (Gough & Whitehall, 1962), (Stewart, 1965).

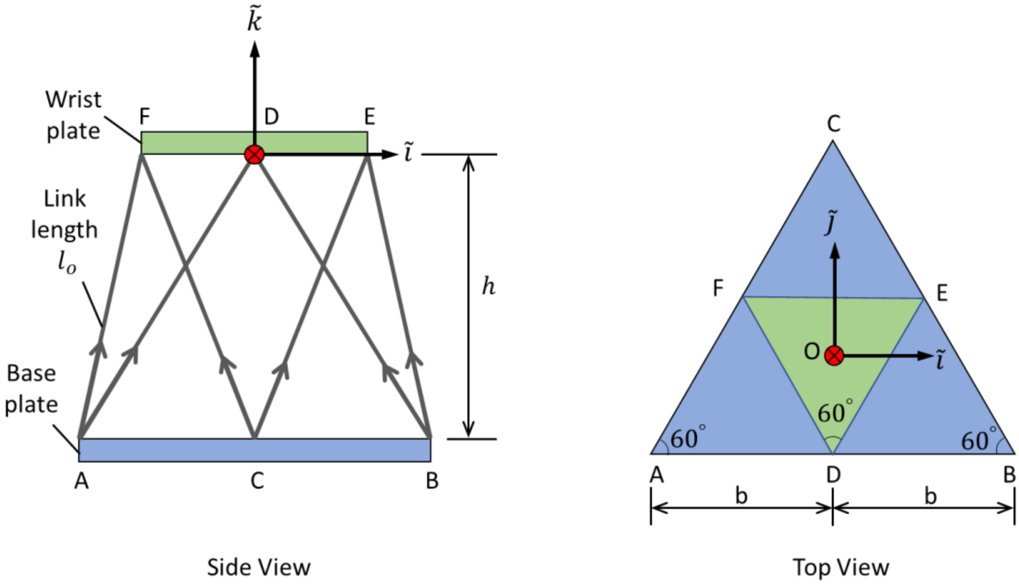

Figure 0-1. Simplified geometry of the compliant device (McCallion, et al., 1979).

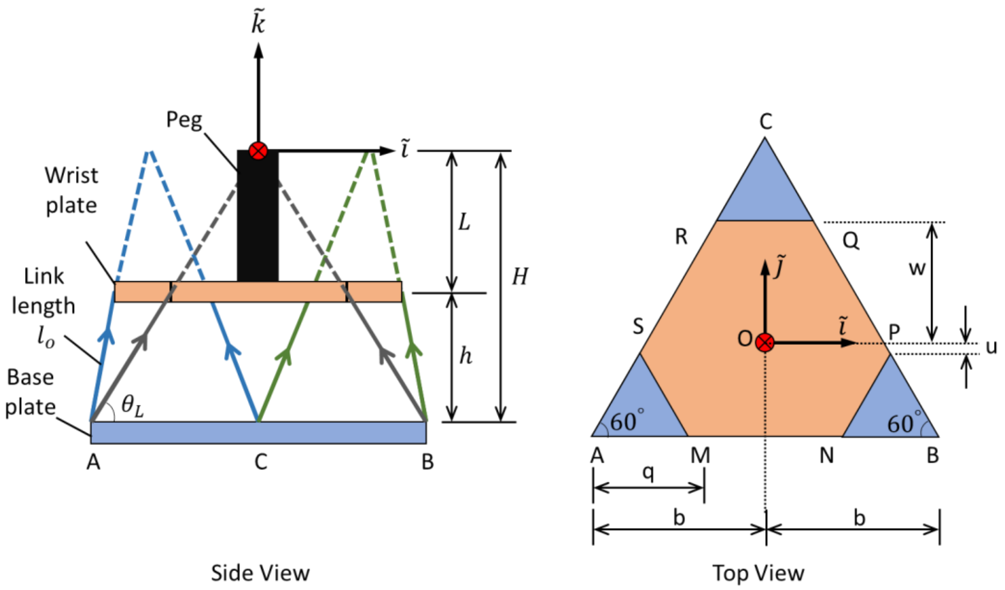

Figure 0-2. Simplified geometry of the new device.

The main difference between the compliant device from (McCallion, et al., 1979) and the new device is that the wrist plate has been lowered by L, which makes the centre of rotation remote, and the tip of the peg will be the centre compliance. In the new device, there are six edges/joints instead of three on the wrist plate.

With this model, all joints between two links are at the same point, but this is not the case in practice. It is also assumed that all joints are frictionless, there is no bending moment acting on the links, and the compressive/tensile forces act along the links. Similar to (McCallion, et al., 1979), this model is only recommended for small translation and rotational errors, and this is dependent on the dimension of the end effector of the robot/machine and the size of the peg. The stiffnesses of all links are considered to be the same.

The new device has six degrees of freedom, which is derived in the appendix. The flexibility matrix [C] of the device about a set of orthogonal axes $(\tilde{\imath}, \tilde{\jmath}, \tilde{k})$ is approximated to a diagonal matrix as follows:

$$\begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} C_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} \tag{0.1}$$

where x, y, z are the small translations and $\theta_x, \theta_y, \theta_z$ are the small rotations along and about $(\tilde{\imath}, \tilde{\jmath}, \tilde{k})$; C_{mm}'s are the flexibility constants, which depend on the dimension of the device and the stiffness of the springs; and $F_x, F_y, F_Z, M_x, M_y, M_z$ are the forces and moments projected in $(\tilde{\imath}, \tilde{\jmath}, \tilde{k})$.

This model is simplified and can be easily applied. Although the derivation is shorter than the actual calculation, it is still able to produce a good approximation that is close to the real values.

The fixed variables are divided into two tiers:

a. Primary fixed variables, which are decided:

$$\begin{gathered} Base\ length, b \\ Peg\ length, L \\ Link's\ tilting\ angle, \theta_L \\ Spring\ constant, k \end{gathered} \tag{0.2}$$

b. Secondary fixed variables, which are calculated based on the primary fixed variable:

$$H = b \times \tan\theta_L \tag{0.3}$$

$$h = H - L \tag{0.4}$$

$$l_o = \frac{h}{\sin \theta_L} \tag{0.5}$$

$$q = \frac{h}{\tan \theta_L} \tag{0.6}$$

$$u = \frac{q\sqrt{3}}{2} - \frac{b\sqrt{3}}{3} \tag{0.7}$$

$$w = \frac{4b\sqrt{3} \; - \; 3q\sqrt{3}}{6} \tag{0.8}$$

However, the manipulated variables would be the translation and rotation $x, y, z, \theta_x, \theta_y, and \; \theta_z$. Then, $F_x, F_y, F_Z, M_x, M_y, M_z$ are the responding variables. This would better describe the effects of misalignments on the device. The model results will then be compared to the numerical results.

The coordinates about the global axes are listed below:

$$OA = \left(\quad -b \quad , \quad \frac{-b\sqrt{3}}{3} \quad , \quad -H \quad \right)$$

$$OB = \left(\quad b \quad , \quad \frac{-b\sqrt{3}}{3} \quad , \quad -H \quad \right)$$

$$OC = \left(\quad 0 \quad , \quad \frac{2b\sqrt{3}}{3} \quad , \quad -H \quad \right)$$

$$OM = \left(\quad -(b - q) \quad , \quad \frac{-b\sqrt{3}}{3} \quad , \quad -L \quad \right)$$

$$ON = \left((b-q) \quad , \quad \frac{-b\sqrt{3}}{3} \quad , \quad -L \right)$$

$$OP = \left((b-\frac{q}{2}) \quad , \quad u \quad , \quad -L \right)$$

$$OQ = \left(\frac{q}{2} \quad , \quad w \quad , \quad -L \right)$$

$$OR = \left(-\frac{q}{2} \quad , \quad w \quad , \quad -L \right)$$

$$OS = \left(-(b-\frac{q}{2}) \quad , \quad u \quad , \quad -L \right)$$

Link 1: $\overrightarrow{l_1} = \overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA}$

Link 2: $\overrightarrow{l_2} = \overrightarrow{BN} = \overrightarrow{ON} - \overrightarrow{OB}$

Link 3: $\overrightarrow{l_3} = \overrightarrow{BP} = \overrightarrow{OP} - \overrightarrow{OB}$

Link 4: $\overrightarrow{l_4} = \overrightarrow{CQ} = \overrightarrow{OQ} - \overrightarrow{OC}$

Link 5: $\overrightarrow{l_5} = \overrightarrow{CR} = \overrightarrow{OR} - \overrightarrow{OC}$

Link 6: $\overrightarrow{l_6} = \overrightarrow{AS} = \overrightarrow{OS} - \overrightarrow{OA}$

The method to find the flexibility matrix [C] is similar to (McCallion, et al., 1979):

1. The local stiffness matrix [s] of the device is found by using the coordinate system listed above, and the local displacement vector, $\overrightarrow{\Delta l}$, is defined. Since the forces act along the links, the local coordinate system is used instead.

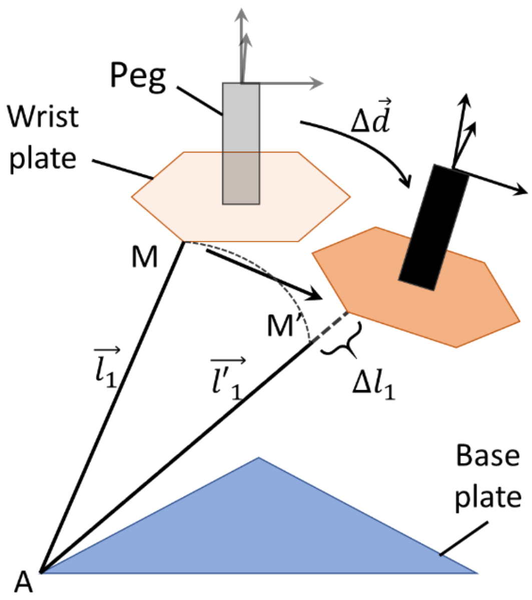

Figure 0-3. Link vectors and the displacement of the wrist plate.

The local stiffness matrix of the device is shown below:

$$[s] = k[I] \tag{0.9}$$

where k is the spring constant and $[I]$ is the 6×6 identity matrix.

The local displacement Δl_m is the small change in the length of link m caused by the axial link forces; l_m is the length of the link and is the link vector. Then, assuming the translation and rotation, $\left(\overrightarrow{l_m}' - \overrightarrow{l_m}\right)$, the change is small, and the relationship below is established.

$$l_m{}^2 = \overrightarrow{l_m}^2 \tag{0.10}$$

$$l_m\,\Delta l_m \approx \overrightarrow{l_m} \cdot \left(\overrightarrow{l_m}' - \overrightarrow{l_m}\right) \tag{0.11}$$

where $\overrightarrow{l_m}'$ is the link vector of link m after its length has changed by Δl_m. Since all links have the same initial length,

$$l_m = l_o \tag{0.12}$$

$$\Delta l_m \approx \frac{\overrightarrow{l_m} \cdot \left(\overrightarrow{l_m}' - \overrightarrow{l_m}\right)}{l_o} \tag{0.13}$$

2. The compatibility matrix [a] is derived from the displacements of joints in the local and global coordinate systems. Let $\Delta\vec{d}$ be the displacement of the wrist plate in the global coordinate system, which is similar to Equation (0.14).

$$\Delta\vec{d} = \begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} \tag{0.14}$$

$$\Delta\vec{l} = \begin{bmatrix} \Delta\widetilde{l_1} \\ \Delta\widetilde{l_2} \\ \Delta\widetilde{l_3} \\ \Delta\widetilde{l_4} \\ \Delta\widetilde{l_5} \\ \Delta\widetilde{l_6} \end{bmatrix} \tag{0.15}$$

$$\Delta\vec{l} = [\mathrm{a}]\,\Delta\vec{d} \tag{0.16}$$

Using Equation (0.13) and solving $\overrightarrow{l_m}'$ as linear functions of $(x, y, z, \theta_x, \theta_y, \theta_z)$, the coefficient a_{mn} can be obtained. Using Link 1, an example of the calculation for obtaining the first row of $[a]$ is shown below.

$$\vec{l_1'} = \overrightarrow{OM'} - \overrightarrow{OA} \tag{0.17}$$

where $\overrightarrow{OM'}$ is the vector of joint M on the wrist plate after translation and rotation,

$$\overrightarrow{OM'} = [R]\,\overrightarrow{OM} + [\vec{t}] \tag{0.18}$$

$$[R] = \begin{bmatrix} 1 & -\theta_z & \theta_y \\ \theta_z & 1 & -\theta_x \\ -\theta_y & \theta_x & 1 \end{bmatrix} \tag{0.19}$$

$$[\overrightarrow{OM}] = \begin{bmatrix} -(b-q) \\ \frac{-b\sqrt{3}}{3} \\ -L \end{bmatrix} \tag{0.20}$$

$$[\vec{t}] = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{0.21}$$

$$[\overrightarrow{OA}] = \begin{bmatrix} -b \\ \frac{-b\sqrt{3}}{3} \\ -H \end{bmatrix} \tag{0.22}$$

Then, substituting Equations (0.18) to (0.22) into Equation (0.17) yields:

$$\vec{l_1'} = \begin{bmatrix} q + \frac{b\sqrt{3}}{3}\theta_z - L\theta_y + x \\ -(b-q)\theta_z + L\theta_x + y \\ (b-q)\theta_y - \frac{b\sqrt{3}}{3}\theta_z + z + h \end{bmatrix} \tag{0.23}$$

The vector $\overrightarrow{l_1}$ is the link between Joints A and M,

$$\overrightarrow{l_1} = \overrightarrow{OM} - \overrightarrow{OA}$$

$$= \begin{bmatrix} -(b-q) \\ \frac{b\sqrt{3}}{3} \\ -L \end{bmatrix} - \begin{bmatrix} -b \\ \frac{b\sqrt{3}}{3} \\ -H \end{bmatrix} \tag{0.24}$$

$$= \begin{bmatrix} q \\ 0 \\ h \end{bmatrix}$$

Using Equation (0.13),

$$\overrightarrow{l_1'} - \overrightarrow{l_1} = \begin{bmatrix} q + \frac{b\sqrt{3}}{3}\theta_z - L\theta_y + x \\ -(b-q)\theta_z + L\theta_x + y \\ (b-q)\theta_y - \frac{b\sqrt{3}}{3}\theta_x + z + h \end{bmatrix} - \begin{bmatrix} q \\ 0 \\ h \end{bmatrix} \tag{0.25}$$

$$= \begin{bmatrix} \frac{b\sqrt{3}}{3}\theta_z - L\theta_y + x \\ -(b-q)\theta_z + L\theta_x + y \\ (b-q)\theta_y - \frac{b\sqrt{3}}{3}\theta_x + z \end{bmatrix}$$

$$\overrightarrow{l_1} \cdot \left(\overrightarrow{l_1'} - \overrightarrow{l_1}\right) = \left(\frac{bq\sqrt{3}}{3}\theta_z - Lq\theta_y + qx + h(b-q)\theta_y - \frac{bh\sqrt{3}}{3}\theta_x + hz\right) \tag{0.26}$$

Substituting Equation (0.26) into Equation (0.13),

$$\Delta l_1 \approx \frac{1}{l_o}\left(\frac{bq\sqrt{3}}{3}\theta_z - Lq\theta_y + qx + h(b-q)\theta_y - \frac{bh\sqrt{3}}{3}\theta_x + hz\right) \tag{0.27}$$

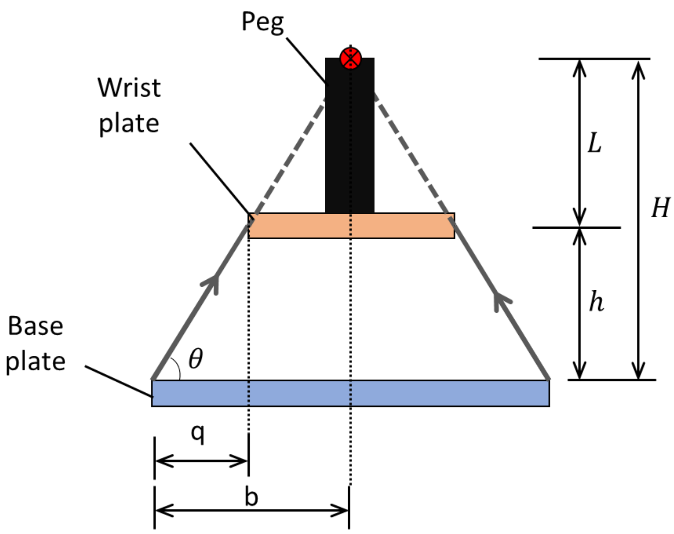

Figure 0-4. Relationship between geometry.

Note that

$$\tan\theta = \frac{H}{b} = \frac{h}{q}$$

$$Hq = hb$$

Therefore, the first line of $[a]$ is

$$\begin{bmatrix} q & 0 & h & -\frac{bh\sqrt{3}}{3} & 0 & \frac{bq\sqrt{3}}{3} \end{bmatrix} \tag{0.28}$$

Repeating this process for the five other links, the entire matrix $[a]$ is:

$$[a] = \begin{bmatrix} q & 0 & h & -\frac{bh\sqrt{3}}{3} & 0 & \frac{bq\sqrt{3}}{3} \\ -q & 0 & h & -\frac{bh\sqrt{3}}{3} & 0 & -\frac{bq\sqrt{3}}{3} \\ -\frac{q}{2} & \frac{q\sqrt{3}}{2} & h & \frac{bh\sqrt{3}}{6} & \frac{-bh}{2} & \frac{bq\sqrt{3}}{3} \\ \frac{q}{2} & -\frac{q\sqrt{3}}{2} & h & \frac{bh\sqrt{3}}{6} & \frac{-bh}{2} & -\frac{bq\sqrt{3}}{3} \\ -\frac{q}{2} & -\frac{q\sqrt{3}}{2} & h & \frac{bh\sqrt{3}}{6} & \frac{bh}{2} & \frac{bq\sqrt{3}}{3} \\ \frac{q}{2} & \frac{q\sqrt{3}}{2} & h & \frac{bh\sqrt{3}}{6} & \frac{bh}{2} & -\frac{bq\sqrt{3}}{3} \end{bmatrix} \tag{0.29}$$

3. The global stiffness matrix $[S]$ is established through the relationship below, where $[a]^T$ is the transpose matrix of $[a]$.

$$[S] = [a]^T\,[s]\,[a] \tag{0.30}$$

Substituting Equations (0.9) and (0.29) into Equation(0.30),

$$[S] = \frac{k}{l_o^2}\begin{bmatrix} 3q^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3q^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6h^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & b^2h^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & b^2h^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2b^2q^2 \end{bmatrix} \tag{0.31}$$

4. The flexibility matrix $[C]$ is the inverse of the global stiffness matrix $[S]$.

$$[C] = \frac{l_o^2}{k}\begin{bmatrix} \frac{1}{3q^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3q^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6h^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{b^2h^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{b^2h^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2b^2q^2} \end{bmatrix} \tag{0.32}$$

Finally, substituting Equation (0.32) into Equation (0.1),

$$\begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} = \frac{l_o^2}{k}\begin{bmatrix} \frac{1}{3q^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3q^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6h^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{b^2h^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{b^2h^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2b^2q^2} \end{bmatrix}\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} \tag{0.33}$$