A passive compliant Gough-Whitehall-Stewart mechanism for peg-hole disassembly : Numerical Validation Model

To validate the approximation model, a set of numerical models is derived. The change in the position of joints in the global coordinate system can be calculated because the position of the joints with respect to the remote centre is known and the manipulated translations and rotations of the remote centre are also known.

The base plate is assumed to be fixed, and the wrist plate is the one that moves. Let θ be the rotation introduced. Using Link 1 as an example, the translation and rotation of Joint M are calculated as follows:

$$\overrightarrow{\mathrm{OM}'} = [R_z(\theta_z)] \cdot \left[R_y(\theta_y)\right] \cdot [R_x(\theta_x)] \cdot \overrightarrow{OM} + [\vec{t}]$$

$$= \begin{bmatrix} \cos\theta_y \cos\theta_z & \cos\theta_z \sin\theta_x \sin\theta_y - \cos\theta_x \sin\theta_z & \sin\theta_x \sin\theta_z + \cos\theta_x \cos\theta_z \sin\theta_y \\ \cos\theta_y \sin\theta_z & \cos\theta_x \cos\theta_z + \sin\theta_x \sin\theta_y \sin\theta_z & \cos\theta_x \sin\theta_y \sin\theta_z - \cos\theta_z \sin\theta_x \\ -\sin\theta_z & \cos\theta_y \sin\theta_x & \cos\theta_x \cos\theta_y \end{bmatrix} \begin{bmatrix} -(b-q) \\ \dfrac{-b\sqrt{3}}{3} \\ -L \end{bmatrix} + \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (0.1)$$

$$[R_x(\theta_x)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix} \quad (0.2)$$

$$\left[R_y(\theta_y)\right] = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix} \quad (0.3)$$

$$[R_z(\theta_z)] = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (0.4)$$

Using Equation **Error! Reference source not found.**, the change in distance between the two points can be calculated.

The force acting along the link is calculated using the equation below and resolved in $(\tilde{\imath}, \tilde{\jmath}, \tilde{k})$ by using the link vector.

$$F_1 = k \times \Delta l_1 \tag{0.5}$$

where

$$\Delta l_1 = l_1' - l_o \tag{0.6}$$

Let $\overrightarrow{AM'}$ be the vector of Link 1 after translation/rotation,

$$\overrightarrow{AM'} = \begin{bmatrix} AM'_x \\ AM'_y \\ AM'_z \end{bmatrix} \tag{0.7}$$

and the length of Link 1,

$$\|AM'\| = \sqrt{AM_x^2 + AM_y^2 + AM_z^2} \tag{0.8}$$

To find the angle, ω, of vector Link 1 with respect to the axes, the method below is used.

$$\begin{aligned} \omega_x &= \cos^{-1}\frac{AM_x}{\|AM'\|} \\ \omega_y &= \cos^{-1}\frac{AM_y}{\|AM'\|} \\ \omega_z &= \cos^{-1}\frac{AM_z}{\|AM'\|} \end{aligned} \tag{0.9}$$

Projecting the force along Link 1 onto the axes,

$$F_{1x} = F_1 \cos \omega_x$$

$$F_{1y} = F_1 \cos \omega_y \tag{0.10}$$

$$F_{1z} = F_1 \cos \omega_z$$

The process is repeated for the rest of the links. The sum of all the projected forces of each axis can now be compared with the approximation model for translation.

However, to calculate the moments caused by rotation, there is an extra step. The moments are also resolved in $(\tilde{\imath}, \tilde{\jmath}, \tilde{k})$ using the equation below:

$$\begin{aligned} \vec{M} &= \vec{r} \times \vec{\mathrm{F}} \\ &= \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \end{aligned} \tag{0.11}$$

$$M_x = r_y F_z - r_z F_y$$

$$M_y = r_z F_x - r_x F_z \tag{0.12}$$

$$M_z = r_x F_y - r_y F_x$$

$\vec{M}$ is the moment,

$\vec{r}$ is the vector between the remote centre to the joint on the Wrist Plate, and

$\vec{F}$ is the force vector, which is already calculated using Equation (0.10).

The calculation is repeated for the rest of the links, summing all the projected moments of each axis and comparing it with the approximation model for rotation.