Appendix A

Algorithms

We used a rather typical Mamory-Based Learning (MBL; see, for example, Haykin, 1999) implementation, k-Nearest Neighbors (kNN) using Euclidean distances. It has been shown that this simple principle straightforwardly ensures the smallest probability of error over all possible decisions for a new exemplar (as discussed in Cover & Hart, 1967). The neighborhood size was set to $k = 7$, following previous computational experimentation (Milin, Keuleers, & Đurđević, 2011). All MBL simulations we made in the **R** software environment (R Core Team, 2021) using the **class** package (Venables & Ripley, 2013).

Two Error-Correction Learning (ECL), Widrow-Hoff (WH; Widrow & Hoff, 1960; Milin, Madabushi, Croucher, & Divjak, 2020) and Temporal Difference (TD; Sutton & Barto, 1987, 1990), were implemented in the **MATLAB** programming and numeric computing environment (MATLAB, 2022). In particular, at time step t, learning assumes small changes to the input weights w_t (small to obey the Principle of Minimal Disturbance; cf., Milin et al., 2020) to improve current state of "knowledge" in the next time step $t + 1$. This is achieved by re-weighting the *prediction error* (which is the difference between the targeted outcome and its current apprehension) with the current input cues and a *learning rate* (free parameter): $\lambda(\boldsymbol{o}_t - \boldsymbol{w}_t\boldsymbol{c}_t)\boldsymbol{c}_t$, where λ represents the learning rate, $\boldsymbol{c}_t$ is the vector of input cues, $\boldsymbol{w}_t\boldsymbol{c}_t$ is the apprehension and $\boldsymbol{o}_t$ is the true outcome. The full WH update of weights w in the next time step $t + 1$ is then:

$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t + \lambda(\boldsymbol{o}_t - \boldsymbol{w}_t\boldsymbol{c}_t)\boldsymbol{c}_t$$

For TD, however, the apprehension ($\boldsymbol{w}_t\boldsymbol{c}_t$) is additionally corrected by a future apprehension as the (temporal) difference to that current apprehension: $\boldsymbol{w}_t\boldsymbol{c}_t - \gamma\boldsymbol{w}_t\boldsymbol{c}_{t+1}$, where γ is a *discount factor*, the additional free parameter that weights the importance of the future, where $\boldsymbol{w}_t\boldsymbol{c}_{t+1}$ is the apprehension given the future – i.e., the next input $\boldsymbol{c}_{t+1}$. The update rule for the TD thus becomes:

$$\begin{aligned}\boldsymbol{w}_{t+1} &= \boldsymbol{w}_t + \lambda(\boldsymbol{o}_t - [\boldsymbol{w}_t\boldsymbol{c}_t - \gamma\boldsymbol{w}_t\boldsymbol{c}_{t+1}])\boldsymbol{c}_t \\ &= \boldsymbol{w}_t + \lambda(\boldsymbol{o}_t + \gamma\boldsymbol{w}_t\boldsymbol{c}_{t+1} - \boldsymbol{w}_t\boldsymbol{c}_t)\boldsymbol{c}_t\end{aligned}$$

Using two closely related ECL algorithms, WH and TD, allows for a detailed examination of the effect of the future apprehension of the TD rule, by explicitly making use of that future data (i.e., $\boldsymbol{w}_t\boldsymbol{c}_t - \gamma\boldsymbol{w}_t\boldsymbol{c}_{t+1}$). Note, however, that if we set the discount factor γ to zero, the whole correction for future predictions $\gamma\boldsymbol{w}_t\boldsymbol{c}_{t+1}$ becomes zero too, and TD becomes identical to WH.

Cover, T., & Hart, P. (1967). Nearest neighbor pattern classification. *IEEE transactions on information theory, 13*(1), 21-27.

Haykin, S. S. (1999). Neural networks: A comprehensive foundation. In (2nd ed.). London: Prentice Hall.

MATLAB. (2022). MATLAB (Version R2022a; 9.12.0.1884302). Natick, Massachusetts: The MathWorks. Retrieved from https://www.mathworks.com/

Milin, P., Keuleers, E., & Đurđević, D. (2011). Allomorphic responses in Serbian pseudo-nouns as a result of analogical learning. *Acta Linguistica Hungarica, 58*(1), 65-84.

Milin, P., Madabushi, H. T., Croucher, M., & Divjak, D. (2020). Keeping it simple: Implementation and performance of the proto-principle of adaptation and learning in the language sciences. *arXiv preprint arXiv:2003.03813*.

R Core Team. (2021). R: A language and environment for statistical computing (Version 4.1.1). Vienna, Austria: R Foundation for Statistical Computing. Retrieved from https://www.R-project.org/

Sutton, R. S., & Barto, A. G. (1987). *A temporal-difference model of classical conditioning.* Paper presented at the Proceedings of the ninth annual conference of the cognitive science society.

Sutton, R. S., & Barto, A. G. (1990). *Time-derivative models of Pavlovian reinforcement.* Paper presented at the Learning and Computational Neuroscience: Foundations of Adaptive Networks.

Venables, W. N., & Ripley, B. D. (2013). *Modern applied statistics with S-PLUS*: Springer Science & Business Media.

Widrow, B., & Hoff, M. E. (1960). *Adaptive switching circuits*. Retrieved from